Answers. Unit One.

Exercise 1A. Page 19.

1. At least one of the seven questions will be done by two or more of the eight students.

2. One of the year three classes will have at least two of the Singh triplets.

3. At least one of the variations of the genetic marker is possessed by more than one human.

4. At least two of the socks will be of the same colour.

5. There are people in Australia who have the same number of hairs on their head as do other people in Australia.

6. Some people who have existed occupy more than one space on my ancestral tree. I.e. some great great great ... grandfather on my mother's side was also a great great great ... grandfather on my father's side.

7. (a) 14

(b) 0

(c) No. If some person A shook hands with all 14 others then none of the other 14 could have shaken hands with nobody because they all at least shook hands with person A.
Similarly if some person shook hands with none of the others then no one could have shaken hands with everyone.

Hence there are 15 people to either assign to the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13
or to assign to the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14.
Either way we have 15 people to assign to 14 integers so at least two people will have shaken hands with the same number of people.

8. If a polygon is a triangle then the polygon has exactly three sides. True. $P \Leftrightarrow Q$.

9. If Jenny's mouth is open then she is talking. False. $P \not\Leftrightarrow Q$.

10. If the animal is a mammal then it is a platypus. False. $P \not\Leftrightarrow Q$.

11. If the car will not start it is out of fuel. False. $P \not\Leftrightarrow Q$.

12. If points are collinear then they lie on the same straight line. True. $P \Leftrightarrow Q$.

13. If tomorrow is not Friday then today is not Thursday. True

14. If a number is not a multiple of two then it is not even. True.

15. If a triangle does not have three different length sides then it is not scalene. True.

16. If my lawn is not wet then my sprinklers are not on. True.

17. If Armand does get up before 8 am then it is a school day. True

18. (a) True
(b) If a polygon is not a triangle then its angles do not add up to 180°. (c) True

19. (a) True
(b) If a positive integer does not have exactly 2 factors then it is not a prime number. (c) True

20. (a) True
(b) If the car battery is not flat then the car will start. (c) False

21. (a) True
(b) If there are no letters in my mail box the post person has not been to our road. (c) False

22. (a) False
(b) If a number is not even then it is not a multiple of 4. (c) True

23.	Converse:	If a polygon is a pentagon then the polygon is five sided.	True.
	Inverse:	If a polygon is not five sided then the polygon is not a pentagon.	True.
	Contrapostive:	If a polygon is not a pentagon then the polygon is not five sided.	True.
24.	Converse:	If the four angles of a quadrilateral are all right angles then the quadrilateral is a square.	False.
	Inverse:	If a quadrilateral is not a square then the four angles of the quadrilateral are not all right angles.	False.
	Contrapostive:	If the four angles of a quadrilateral are not all right angles then the quadrilateral is not a square.	True

Miscellaneous Exercise One. Page 21.

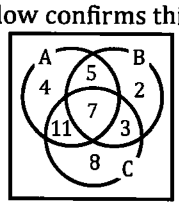
1. The ladder will make an angle of $71°$ with the ground (to nearest degree).

Exercise 2A. Page 27.

1. (a) 6 (b) 8 (c) 120 (d) 11 (e) 110
(f) 15 (g) 20 (h) 210 (i) 56
2. 6
3. 16
4. 719
5. 243
6. (a) 5 040 (b) 823 543
7. (a) 2 520 (b) 16 807
8. (a) 3 375 (b) 2 730
9. (a) 57 600 (b) 12 441 600 (c) 311 040 000
10. 132
11. 5 040. PIN: **P**ersonal **I**dentification **N**umber
12. 665 280
13. 1 048 576
14. 2 730
15. 336, 40 320
16. 1 024

Exercise 2B. Page 32.

1. 360
2. 840
3. 420
4. 239 500 800
5. 34650, 3150
6. 75600, 7560, 68040
7. 80
8. 18 252
9. 16 250
10. (a) 36 (b) 12
11. (a) 150 (b) 80
12. 1 465
13. 90
14. 468
15. (a) 33 280 (8^5 long key base + 8^3 short key base)
(b) 7 056 (6 720 long key base + 336 short key base)
16. (a) 59 778 (9^5 long key base + 9^3 short key base)
(b) 15 624 (15 120 long key base + 504 short key base)
17. 36
18. 1200
19. 79
20. 30
21. 78
22. (a) 199 (b) 142 (c) 313
23. $n(A \cup B \cup C) = 40$
Venn diagram below confirms this answer of 40.



24. 74
25. 413
26. $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D|$
$- |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D|$
$- |A \cap B \cap C \cap D|$

Exercise 2C. Page 37.

1. (a) 24 (b) 625
2. (a) 360 (b) 72
3. (a) 720 (b) 240 (c) 24 (d) 144
4. 120 (a) 72 (b) 48
5. 120 (a) 24 (b) 24 (c) 6
6. (a) 5 040 (b) 2 160 (c) 360
7. (a) 24 (b) 6 (c) 3
8. (a) 486 720 (b) 650 000 (c) 421 200 (d) 117 000
9. (a) 144 (b) 24 (c) 72
10. (a) 1 757 600 (b) 1 404 000 (c) 1 134 000 (d) 6 500 (e) 2 400 (f) 216
11. (a) 3 628 800 (b) 40 320 (c) 241 920 (d) 5 040

Exercise 2D. Page 43.

1. (a) 750 (b) 180 (c) 108
2. (a) 7992 (b) 840 (c) 700
3. (a) 2160 (b) 600
4. 120 (a) 24 (b) 24 (c) 6 (d) 42
5. (a) 40320 (b) 10080 (c) 30240 (d) 1440 (e) 9360
6. (a) 210 (b) 30 (c) 30 (d) 5 (e) 55 (f) 120 (g) 30 (h) 90 (i) 40
7. (a) 6 (b) 6 (c) 2 (d) 10 (e) 4 (f) 12
8. (a) 70560 (b) 25200
9. 29030400
10. (a) 130 (b) 26 (c) 5 (d) 1 (e) 30 (f) 5 (g) 125

Exercise 2E. Page 51.

1. In a combination lock the order of the numbers is important. Thus to be more correct it should really be called a permutation lock. Hence a combination lock is not correctly named.
2. 10
3. 4845
4. 4200
5. 700
6. 36
7. 495, 240
8. 128
9. 510
10. 163800 (a) 13650 (b) 65520 (c) 73710
11. (a) 210 (b) 28 (c) 98 (d) 182
12. (a) 1400 (b) 8 (c) 0 (d) 176 (e) 1016
13. (a) 752538150 (b) 115775100 (c) 171028000 (d) 73629072 (e) 80672868
14. 715, 360
15. 70, 22
16. 399
17. 5472

Miscellaneous Exercise Two. Page 55.

1. 39·7
2. 8·4
3. Converse: If $x^2 = 64$ then $x = 8$. False.
   Contrapositive: If $x^2 \neq 64$ then $x \neq 8$. True.
4. There are 24 different permutations ($4 \times 3 \times 2 \times 1$) and twenty five responses from the students. Hence, by the pigeon hole principle, at least two pieces of paper will feature the same permutation.
5. There are 44352 different bets.
6. 15120, 7200
7. 15504, 3072

Exercise 3A. Page 59.

1. (a) 11·5 km, 071° (b) 251°
2. (a) 5·5 km, 027° (b) 207°
3. (a) 47 km, 304° (b) 124°
4. (a) 1150 m, 089° (b) 269°
5. (a) 87 m, 046° (b) 226°
6. (a) 66 km, 235° (b) 055°
7. Approximately 41 m.
8. 8·9 km, 145°
9. 3·2 km, 095°
10. 071°, 349°
11. 286 m in direction 060°.

Exercise 3B. Page 63.

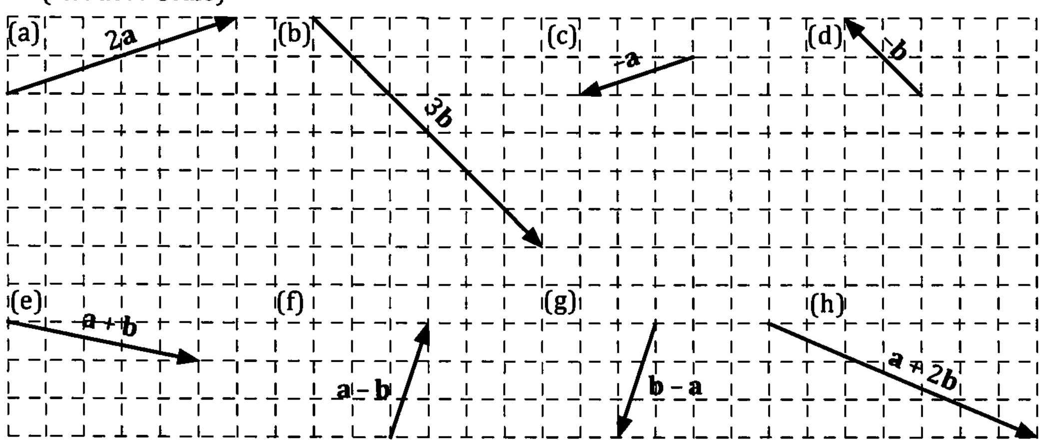
1. 8·3 N at 27° to the vertical.
2. 16·3 N at 22° to the vertical.
3. 28·3 N at 0° to the vertical.
4. 24·4 N at 25° to the vertical.
5. $5\sqrt{3}$ N, 090°
6. $2\sqrt{31}$ N, 159°
7. 12·1 N, 018°
8. 11·7N, 158°
9. 47 N at 66° to the slope.
10. 90 N at 78° to the slope.
11. 38 N at 67° to the slope.
12. ~9·2 N at ~42° to the larger force.
13. ~23·2 N at ~27° to the smaller force.

Exercise 3C. Page 65.

1. 4·5 m/s at 63° to the bank.
2. 3·6 m/s at 85° to the bank.
3. 5·5 m/s at 34° to the bank
4. 353°. Approximately 15·3 km
5. 170° at 72 km/h, 194°
6. (a) 180 m (b) $\sqrt{10}$ m/s (≈ 3·2 m/s) (c) ~72°
7. (a) Upstream at 73° to the bank, 30 seconds.
   (b) Upstream at 66° to the bank, 31 seconds.
   (c) Upstream at 53° to the bank, 36 seconds.
8. 356°
9. 005°
10. 048°, 1hr 34 mins, 1hr 20 mins
11. 46 secs (19·1 + 14·0 + 13·3)

Exercise 3D. Page 72.

1. (a) **d** and **e** (b) **c** and **d** or **c** and **e**
   (c) **a** and **b** or **a** and **f** (d) **b** and **f**
2. (a) $\mathbf{b} + \mathbf{c} = \mathbf{a}$ (b) $\mathbf{a} + \mathbf{b} = \mathbf{c}$ (c) $\mathbf{a} + \mathbf{c} = \mathbf{b}$
3. $\mathbf{b} = \mathbf{a}$, $\mathbf{c} = \frac{1}{2}\mathbf{a}$, $\mathbf{d} = -\mathbf{a}$, $\mathbf{e} = -\frac{1}{2}\mathbf{a}$,
   $\mathbf{f} = -\frac{1}{4}\mathbf{a}$, $\mathbf{g} = \frac{3}{2}\mathbf{a}$, $\mathbf{h} = \frac{3}{4}\mathbf{a}$.
4. $\mathbf{p} = 2\mathbf{m}$, $\mathbf{q} = -\mathbf{n}$, $\mathbf{r} = 2\mathbf{n}$, $\mathbf{s} = -\mathbf{m}$,
   $\mathbf{t} = \frac{1}{2}\mathbf{n}$, $\mathbf{u} = \mathbf{m} + \mathbf{n}$, $\mathbf{v} = \mathbf{m} + 2\mathbf{n}$.
5. $\mathbf{c} = \mathbf{a} + \mathbf{b}$, $\mathbf{d} = \mathbf{a} - \mathbf{b}$, $\mathbf{e} = \mathbf{b} - \mathbf{a}$, $\mathbf{f} = 2\mathbf{b} + \mathbf{a}$, $\mathbf{g} = \mathbf{b} + 2\mathbf{a}$.
6. $\mathbf{u} = \mathbf{s} + \mathbf{t}$, $\mathbf{v} = 2\mathbf{s} + \mathbf{t}$, $\mathbf{w} = \mathbf{s} - \mathbf{t}$, $\mathbf{x} = -\mathbf{s} + \mathbf{t}$, $\mathbf{y} = 2\mathbf{t} + 2\mathbf{s}$, $\mathbf{z} = 3\mathbf{s} + \frac{3}{2}\mathbf{t}$.
7. (Reduced scale)



8. (a) 5·8 units in direction 028° (b) 6·9 units in direction 105°
9. (a) 65 units in direction 151° (b) 91 units in direction 100°
10. 1·9 m/s^2 in direction 235°
11. 4·6 m/s^2 in direction 238°
12. (a) $\lambda = 0,\ \mu = 0$ (b) $\lambda = 0,\ \mu = 0$ (c) $\lambda = 3,\ \mu = -4$
    (d) $\lambda = 2,\ \mu = 5$ (e) $\lambda = 5,\ \mu = -2$ (f) $\lambda = 1,\ \mu = 3$
    (g) $\lambda = 2,\ \mu = -1$ (h) $\lambda = 3,\ \mu = -2$ (i) $\lambda = 1,\ \mu = -3$
    (j) $\lambda = 4,\ \mu = -2$
13. (a) $\mathbf{a}$ (b) $-\mathbf{a}$ (c) $\mathbf{c}$ (d) $-\mathbf{c}$
    (e) $\frac{1}{2}\mathbf{c}$ (f) $\mathbf{c} + \frac{1}{2}\mathbf{a}$ (g) $\mathbf{a} + \frac{1}{2}\mathbf{c}$ (h) $\frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{a}$

14. (a) $\mathbf{b} - \mathbf{a}$ (b) $\frac{3}{4}(\mathbf{b} - \mathbf{a})$ (c) $\frac{1}{4}(\mathbf{b} - \mathbf{a})$ (d) $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

15. (a) $\mathbf{a} + \mathbf{b}$ (b) $\frac{1}{3}\mathbf{b}$ (c) $\frac{1}{2}\mathbf{a}$ (d) $\mathbf{a} + \frac{1}{3}\mathbf{b}$

(e) $\mathbf{b} + \frac{1}{2}\mathbf{a}$ (f) $\mathbf{b} - \frac{1}{2}\mathbf{a}$ (g) $\mathbf{a} - \frac{2}{3}\mathbf{b}$ (h) $\frac{2}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$

16. (a) $\mathbf{a} + \mathbf{b}$ (b) $2\mathbf{b}$ (c) $\mathbf{b} - \mathbf{a}$

(d) $\frac{1}{2}(\mathbf{b} - \mathbf{a})$ (e) $\frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b}$

17. (a) $\frac{1}{2}\mathbf{a}$ (b) $\mathbf{b} - \mathbf{a}$ (c) $\frac{2}{3}(\mathbf{b} - \mathbf{a})$

(d) $\frac{2}{3}\mathbf{b} - \frac{1}{6}\mathbf{a}$ (e) $h = 3$, $k = 2$

18. $h = \frac{3}{2}$, $k = \frac{5}{4}$

Miscellaneous Exercise Three. Page 76.

1. There are 64 different settings for the system. **2.** 337°, 3·4 km

3. 495 **4.** 25

5. Converse: If you attend XYZ high school then you are in my Specialist Mathematics class. False.

Contrapositive: If you do not attend XYZ high school then you are not in my Specialist Mathematics class. True.

6. 259 459 200

7. (a) 18·1 units in direction 121° (b) 12·7 units in direction 222° (b) 29·0 units in direction 109°

8. (a) $h = 0, k = 0$ (b) $h = 0, k = 1$ (c) $h = 3, k = -1$

(d) $h = -5, k = 0$ (e) $h = 1, k = -2$ (f) $h = 4, k = -1$

9. (a) 1 800 (b) 252 (c) 3 312 (d) 1 056

Exercise 4A. Page 82.

Note: In this and future vector exercises the choice as to whether answers are presented as

$a\mathbf{i} + b\mathbf{j}$, $\langle a, b \rangle$ or $\begin{pmatrix} a \\ b \end{pmatrix}$ is determined by the notation used in the question.

1. 14·3 N, 334° **2.** 13·2 m/s, 074° **3.** 10·5 units, 142° **4.** 15·7 N, 318°

5. $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$ $\mathbf{b} = 3\mathbf{i} + \mathbf{j}$ $\mathbf{c} = 2\mathbf{i} + 2\mathbf{j}$ $\mathbf{d} = -\mathbf{i} + 3\mathbf{j}$ $\mathbf{e} = 2\mathbf{j}$ $\mathbf{f} = -\mathbf{i} + 2\mathbf{j}$

$\mathbf{g} = \mathbf{i} - 2\mathbf{j}$ $\mathbf{h} = 4\mathbf{i}$ $\mathbf{k} = 2\mathbf{i} - 4\mathbf{j}$ $\mathbf{l} = 4\mathbf{i} - \mathbf{j}$ $\mathbf{m} = -4\mathbf{i} - \mathbf{j}$ $\mathbf{n} = 9\mathbf{i} + 2\mathbf{j}$

6. $|\mathbf{a}| = \sqrt{13}$ units $|\mathbf{b}| = \sqrt{10}$ units $|\mathbf{c}| = 2\sqrt{2}$ units $|\mathbf{d}| = \sqrt{10}$ units

$|\mathbf{e}| = 2$ units $|\mathbf{f}| = \sqrt{5}$ units $|\mathbf{g}| = \sqrt{5}$ units $|\mathbf{h}| = 4$ units

$|\mathbf{k}| = 2\sqrt{5}$ units $|\mathbf{l}| = \sqrt{17}$ units $|\mathbf{m}| = \sqrt{17}$ units $|\mathbf{n}| = \sqrt{85}$ units

7. 25 Newtons

8. (a) $(4{\cdot}3\mathbf{i} + 2{\cdot}5\mathbf{j})$ units (b) $(3{\cdot}5\mathbf{i} + 6{\cdot}1\mathbf{j})$ units (c) $(9{\cdot}1\mathbf{i} + 4{\cdot}2\mathbf{j})$ units

(d) $(5{\cdot}4\mathbf{i} + 4{\cdot}5\mathbf{j})$ N (e) $(-4\mathbf{i} + 6{\cdot}9\mathbf{j})$ m/s (f) $(9{\cdot}4\mathbf{i} - 3{\cdot}4\mathbf{j})$ N

(g) $(-2{\cdot}6\mathbf{i} + 3{\cdot}1\mathbf{j})$ units (h) $(7{\cdot}3\mathbf{i} - 3{\cdot}3\mathbf{j})$ units (i) $(-4{\cdot}6\mathbf{i} - 3{\cdot}9\mathbf{j})$ units

(j) $(-6{\cdot}4\mathbf{i} + 7{\cdot}7\mathbf{j})$ m/s (k) $(-7{\cdot}3\mathbf{i} - 3{\cdot}4\mathbf{j})$ N (l) $(4{\cdot}1\mathbf{i} + 2{\cdot}9\mathbf{j})$ m/s

9. (a) 5 units, 53·1° (b) $\sqrt{29}$ units, 21·8° (c) $\sqrt{13}$ units, 123·7°

(d) 5 units, 53·1° (e) $\sqrt{41}$ units, 38·7° (f) $4\sqrt{2}$ units, 45°

10. (a) −330 km/h (To nearest 10 km/h.) (b) 120 km/h (To nearest 10 km/h.)

11. $\sqrt{89}$ units in direction 328°

12. (a) $3\mathbf{i} + 7\mathbf{j}$ (b) $\mathbf{i} - \mathbf{j}$ (c) $-\mathbf{i} + \mathbf{j}$ (d) $4\mathbf{i} + 6\mathbf{j}$ (e) $3\mathbf{i} + 12\mathbf{j}$ (f) $7\mathbf{i} + 18\mathbf{j}$
(g) $\mathbf{i} - 6\mathbf{j}$ (h) $-\mathbf{i} + 6\mathbf{j}$ (i) $\sqrt{13}$ units (j) $\sqrt{17}$ units (k) $\sqrt{13} + \sqrt{17}$ units
(l) $\sqrt{58}$ units

13. (a) $4\mathbf{i} - \mathbf{j}$ (b) $-\mathbf{i} - 2\mathbf{j}$ (c) $\mathbf{i} + 2\mathbf{j}$ (d) $5\mathbf{i} - 5\mathbf{j}$ (e) $7\mathbf{i} - 4\mathbf{j}$ (f) $9\mathbf{i} - 3\mathbf{j}$
(g) $12\mathbf{i} + 3\mathbf{j}$ (h) $-3\mathbf{j}$ (i) 3 units (j) $\sqrt{2} + \sqrt{5}$ ($\approx 3{\cdot}65$) units (k) 3 units
(l) $\sqrt{5}$ units

14. (a) $< 7, 1 >$ (b) $< 3, 7 >$ (c) $< 10, 8 >$ (d) $< 17, 9 >$ (e) $< -1, -10 >$ (f) $\sqrt{41}$ units
(g) $5\sqrt{2}$ units (h) $\sqrt{41} + \sqrt{13}$ units

15. (a) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ (c) $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$ (d) $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ (e) $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ (f) $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$
(g) $\sqrt{41}$ units (h) $\sqrt{41}$ units

16. (a) $\sqrt{53}$ units (b) $\sqrt{13}$ units (c) $2\sqrt{53}$ units (d) 10 units (e) $4\sqrt{2}$ units

17. (a) ~3760 N (b) ~1370 N **18.** $(17{\cdot}7\mathbf{i} + 9{\cdot}2\mathbf{j})$ N **19.** $(2{\cdot}3\mathbf{i} + 9{\cdot}2\mathbf{j})$ N

20. $(9{\cdot}2\mathbf{i} + 8{\cdot}6\mathbf{j})$ N **21.** $(5{\cdot}9\mathbf{i} + 3{\cdot}5\mathbf{j})$ m/s **22.** $(16{\cdot}2\mathbf{i} + 3{\cdot}9\mathbf{j})$ N

23. $(10{\cdot}3\mathbf{i} + 1{\cdot}1\mathbf{j})$ N **24.** $2\sqrt{17}$ N **25.** $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j}$

26. $\mathbf{c} = -2\mathbf{i} + 11\mathbf{j}$, $\mathbf{d} = 3\mathbf{i} - 16\mathbf{j}$

Exercise 4B. Page 89.

1. For vector **a** (a) $4\mathbf{i} + 3\mathbf{j}$ (b) $8\mathbf{i} + 6\mathbf{j}$ (c) $\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$ (d) $\frac{8}{5}\mathbf{i} + \frac{6}{5}\mathbf{j}$

For vector **b** (a) $4\mathbf{i} - 3\mathbf{j}$ (b) $8\mathbf{i} - 6\mathbf{j}$ (c) $\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$ (d) $\frac{8}{5}\mathbf{i} - \frac{6}{5}\mathbf{j}$

For vector **c** (a) $2\mathbf{i} + 2\mathbf{j}$ (b) $4\mathbf{i} + 4\mathbf{j}$ (c) $\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$ (d) $\sqrt{2}\,\mathbf{i} + \sqrt{2}\,\mathbf{j}$

For vector **d** (a) $3\mathbf{i} - 2\mathbf{j}$ (b) $6\mathbf{i} - 4\mathbf{j}$ (c) $\frac{3}{\sqrt{13}}\mathbf{i} - \frac{2}{\sqrt{13}}\mathbf{j}$ (d) $\frac{6}{\sqrt{13}}\mathbf{i} - \frac{4}{\sqrt{13}}\mathbf{j}$

2. (a) $\frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}$ (b) $2\sqrt{5}\,\mathbf{i} + \sqrt{5}\,\mathbf{j}$ (c) $-\frac{3\sqrt{13}}{5}\mathbf{i} + \frac{4\sqrt{13}}{5}\mathbf{j}$ (d) $\frac{10}{\sqrt{13}}\mathbf{i} + \frac{15}{\sqrt{13}}\mathbf{j}$

3. (a) **a** and **d** (b) $12\mathbf{i} - 14\mathbf{j}$ (c) $2\sqrt{85}$ units (d) $139°$

4. $w = -4$, $x = 0{\cdot}75$, $y = \pm\frac{\sqrt{3}}{2}$, $z = -9$ or 15.

5. $a = 0{\cdot}8$, $b = 3$, $c = -4$, $d = 5$, $e = -12$, $f = \frac{25}{13}$, $g = -\frac{60}{13}$.

6. 21·3 units, $-9{\cdot}9\mathbf{i} - 18{\cdot}9\mathbf{j}$ **7.** 7·8, 50° **8.** 11·9, 31°

9. 9·5, 36° **10.** $T_1 = T_2 = \frac{100}{\sqrt{3}}$ N

11. $T_1 = T_2 = 100$ N **12.** $T_1 = 50\sqrt{3}$ N, $T_2 = 50$ N

13. Particle A is moving the fastest. **14.** The body will move approximately 323 metres.

15. (a) $75\mathbf{j}$ m/s. Appoximately 67 minutes (b) $(-21\mathbf{i} + 72\mathbf{j})$m/s. Approximately 61 minutes.

16. $(-21\mathbf{i} - 72\mathbf{j})$ m/s. Approximately 81 minutes.

17. $\mathbf{a} = -\mathbf{p}$ $\mathbf{b} = 2\mathbf{p}$ $\mathbf{c} = \mathbf{p} + \mathbf{q}$ $\mathbf{d} = \mathbf{p} + 2\mathbf{q}$ $\mathbf{e} = \mathbf{q} - \mathbf{p}$ $\mathbf{f} = 2\mathbf{p} + \mathbf{q}$
$\mathbf{g} = 3\mathbf{p} + 3\mathbf{q}$ $\mathbf{h} = 3\mathbf{p} + 2\mathbf{q}$ $\mathbf{k} = \mathbf{q} - 3\mathbf{p}$ $\mathbf{l} = 2\mathbf{p} - 2\mathbf{q}$ $\mathbf{m} = \mathbf{q} - 2\mathbf{p}$

18. $(5\mathbf{i} - 5\sqrt{3}\mathbf{j})$ N

19. (a) $\mathbf{a} + \mathbf{b}$ (b) $2\mathbf{a} + \mathbf{b}$ (c) $2\mathbf{a} - 3\mathbf{b}$ (d) $\frac{11}{5}\mathbf{a} - \frac{2}{5}\mathbf{b}$ (e) $\frac{2}{5}\mathbf{a} + \frac{11}{5}\mathbf{b}$ (f) $2\mathbf{a} - \mathbf{b}$

20. $(-112\mathbf{i} + 384\mathbf{j})$ km/h, $(140{\cdot}8\mathbf{i} - 374{\cdot}4\mathbf{j})$ km/h

Exercise 4C. Page 94.

1.	2**i** + 5**j**	−3**i** + 6**j**,	0**i** − 5**j**,	3**i** + 8**j**.		
2.	(a) −**i** − 2**j**	(b) **i** + 2**j**				
3.	(a) 3**i** − 7**j**	(b) −**i** + 8**j**	(c) −2**i** − **j**			
4.	(a) 3**i** − 4**j**	(b) −5**i** + 13**j**	(c) 7**i** − 24**j**	(d) 15**i** − 20**j**		
5.	(a) $\sqrt{58}$ units	(b) $\sqrt{5}$ units	(c) $\sqrt{61}$ units			
6.	(a) 5 units	(b) 5 units	(c) $\sqrt{17}$ units	(d) $2\sqrt{17}$ units		
7.	(a) $\sqrt{37}$ units	(b) $\sqrt{34}$ units	(c) $3\sqrt{5}$ units			
8.	(a) −**i** + 5**j**	(b) 8**i** + 19**j**	(c) −3**i** − 23**j**	(d) 25 units	(e) **i** + 2**j**	(f) 16**i** + 45**j**
9.	10**i** + 3**j**	**10.** (a) **i** + 10**j**	(b) 3**i** + 4**j**	(c) 2**i** − 6**j**		
11.	(a) 3**i** + 5**j**	(b) 4**i** − 3**j**	(c) **i** − 8**j**	(d) 13 units		
12.	(a) (4**i** + 4**j**)m	(b) (6**i** − **j**)m	(c) (22**i** − 41**j**)m	(d) 20 metres		
13.	(a) (7**i** + 6**j**) m	(b) (8**i** + 12**j**)m	(c) (12**i** + 36**j**)m	(d) 26 m	(e) 9	
16.	10**i** + 9**j**	**17.** 2**i** + **j**	**18.** 4·6**i** + 6·8**j**			

Miscellaneous Exercise Four. Page 96.

1. $\lambda = \frac{11}{17}$, $\mu = \frac{4}{17}$

2. 150

3. Converse: If a positive whole number is a multiple of five then the number ends in a five. False.

Contrapositive: If a positive whole number is not a multiple of five then the number does not end in a five. True.

4. There are 95 040 ($= 12 \times 11 \times 10 \times 9 \times 8$, or ${}^{12}C_5 \times 5!$) possible different ordered lists.

5. $\Delta ABC \cong \Delta XWV$ (SAS), $\Delta GHI \cong \Delta BDC$ (SSS), $\Delta MNO \cong \Delta TUS$ (RHS), $\Delta PQR \cong \Delta YZA$ (AAcorresS).

6. a = 0, b = 15

7. (a) 144 (b) 3250

8. (a) 2·4 seconds (b) 1·08 metres

Exercise 5A. Page 102

15. $x = 3$ $y = 13$

Exercise 5B. Page 107

11. $x = 12$ $y = 10$

12. $x = 30$ $y = 5$

Miscellaneous Exercise Five. Page 111.

1. 1 287, 40 320

2. 45

3. 70, 30

4. 166°

5. (a) (−3400**i** − 9400**j**) N (b) 3400 N

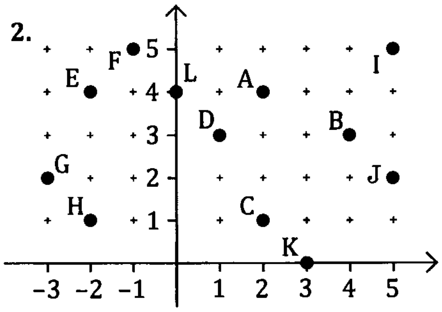
6. Compare your answer with those of others in your class.

7. (a) 2**b** (b) $\frac{4}{3}$ **b** (c) **a** + **b** (d) **a** + 2**b** (e) $\frac{\mathbf{a}+3\mathbf{b}}{2}$ (f) $\frac{3\mathbf{a}-\mathbf{b}}{6}$, $h = \frac{3}{7}$, $k = \frac{2}{7}$.

Exercise 6A. Page 117.

1. (a) $2\mathbf{i} + 3\mathbf{j}$ (b) $4\mathbf{i} + 5\mathbf{j}$ (c) $\mathbf{i} + 4\mathbf{j}$
(d) $-2\mathbf{i} - 2\mathbf{j}$ (e) $\mathbf{i} - \mathbf{j}$ (f) $-2\mathbf{i} + 2\mathbf{j}$
(g) $2\mathbf{i} - 2\mathbf{j}$ (h) $3\mathbf{i} + \mathbf{j}$ (i) $-3\mathbf{i} + 3\mathbf{j}$
(j) $3\mathbf{i} - 3\mathbf{j}$

2.



3. $(-5\mathbf{i} + 3\mathbf{j})$ km
4. B relative to A
5. $\sqrt{386}$ km
6. $\sqrt{10}$ km
7. $5\mathbf{i} - \mathbf{j}$

Exercise 6B. Page 121.

1. $6\mathbf{i} - 10\mathbf{j}$
2. $-3\mathbf{i} + 3\mathbf{j}$
3. $-\mathbf{i} - 9\mathbf{j}$
4. $3\mathbf{i} + 3\mathbf{j}$
5. $20\sqrt{3}$ km/h in direction 060°.
6. 10 km/h in direction 037°.
7. 15·8 km/h in direction 318°.
8. 28·8 km/h in direction 075°.
9. 11·8 km/h in direction 343°.
10. 17·3 km/h in direction 308°.
11. (a) $(5\mathbf{i} - 30\mathbf{j})$ km/h (b) $(-5\mathbf{i} + 30\mathbf{j})$ km/h
12. (a) 26·1 km/h in direction 253°. (b) 26·1 km/h in direction 073°.
13. 174 km/h in direction 287°.
14. $\mathbf{i} - 12\mathbf{j}$
15. $-2\mathbf{i} + 6\mathbf{j}$
17. 10 km/h due South.
18. 20 km/h due South.
19. 160 km/h, 076°
20. $(13\mathbf{i} + \mathbf{j})$ km/h
21. $(4\mathbf{i} - 2\mathbf{j})$ km/h
22. 17·9 km/h from 279°.
23. $\sqrt{34}$ km/h from 211°.
24. B: 10 km/h due North. C: 7 km/h due North. D: 15 km/h due North.
25. Approximately 10 km/h from 208°.
26. F: at rest, G: 19·1 km/h, 030°, H: 31·1 km/h, 328°.
27. $6\mathbf{i} + 8\mathbf{j}$
28. 14·8 km/h from N27°W.
29. 6·2 km/h from S44°W.

Miscellaneous Exercise Six. Page 124.

1. $\mathbf{p} = 3\mathbf{i} + 3\sqrt{3}\,\mathbf{j}$, $\mathbf{q} = -8\mathbf{i} + 8\mathbf{j}$, $\mathbf{r} = -5\mathbf{i} + 5\sqrt{3}\,\mathbf{j}$, $\mathbf{s} = 4\sqrt{3}\,\mathbf{i} - 4\mathbf{j}$.
2. $x = 5$, $y = \pm 1$
3. (a) 5040 (b) 720 (c) 120
4. Compare your response with that of others in your class.
5. Compare your proof with that of others in your class.
6. (a) R is 10 units from Q.
(b) R has position vector $5\mathbf{i} + 4\mathbf{j}$.
(c) R is $\sqrt{41}$ units from the origin.
7. The contrapositive, if not Q then not P, must also be true.
The other two statements, the converse and the inverse could be true or false.
8. 13 300, 9 310
9. 462, 194
10. (a) 56 (b) Option I: 6, Option II: 20, Option III: 50.

Exercise 7A. Page 128.

2. $\frac{1}{2}$ **b**, $\frac{1}{2}$ (**c** – **a**), $\frac{1}{2}$ **b**, $\frac{1}{2}$ (**c** – **a**).

3. $\frac{1}{2}$ (**a** – **c**), **a** – **c**.

5. (b) The diagonals of a quadrilateral bisect each other ⇔ the quadrilateral is a parallelogram.
The diagonals of a quadrilateral bisect each other if and only if the quadrilateral is a parallelogram.

6. (a) (i) **b** – **a** (ii) $\frac{1}{2}(\mathbf{b} - 2\mathbf{a})$ (iii) $\frac{1}{2}(\mathbf{b} - \mathbf{a})$
(iv) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ (v) $\frac{1}{3}(\mathbf{a} + \mathbf{b})$ (vi) $\frac{1}{3}(\mathbf{b} - 2\mathbf{a})$

8. $h = \frac{2}{3}$, $k = \frac{2}{3}$. $\lambda = \frac{2}{3}$

10. (a) $\frac{1}{2}m\mathbf{a} - h\mathbf{a} + \frac{1}{2}m\mathbf{b}$ (b) $k\mathbf{b} - \frac{1}{2}m\mathbf{b} - \frac{1}{2}m\mathbf{a}$

Miscellaneous Exercise Seven. Page 131.

1. (a) Now triangle is scalene ⇒ triangle has three different length sides,
and triangle has three different length sides ⇒ triangle is scalene.
Hence triangle is scalene ⇔ triangle has three different length sides.
Thus "triangle has three different length sides" and "triangle is scalene" are equivalent statements and so the "if and only if" phrase can be used. Hence given statement is correct.
(b) Whilst it is true that if a positive whole number ends with a 0 then it is a multiple of five, the converse is false, because a multiple of 5 does not have to end with a 0. Hence this is not an "if and only if" situation. The given statement is incorrect.

2. 336

3. (a) 5005 (b) 720

4. 73256400

5. Compare your proof with those of others in your class.

6. (a) 5·8**i** + 0·2**j** (b) 9**i** – 3**j**

7. Two seconds later the object is $6\sqrt{5}$ metres from the origin.

8. (**a** + **b**) has magnitude $4\sqrt{5}$ units.

9. (a) 1 hr 50 mins (b) 2 hr 13 mins

10. (a) 2**b** – **a** (b) $\frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$ (c) $\frac{5}{3}\mathbf{b} + \frac{2}{3}\mathbf{a}$. $h = 1{\cdot}5$, $k = 0{\cdot}5$.

11. (a) 358800 (b) 456976
(c) 165765600 (d) 308915776, 6 (including A and R to start with).

Exercise 8A. Page 136.

1.	$\frac{15\sqrt{3}}{2}$	**2.**	0	**3.**	–6	**4.**	10	**5.**	$-5\sqrt{3}$
6.	0	**7.**	0	**8.**	0	**9.**	4	**10.**	9
11.	$6\sqrt{2}$	**12.**	$6\sqrt{2}$	**13.**	12	**14.**	–24	**15.**	3
16.	0	**17.**	$-35\sqrt{3}$	**18.**	$-100\sqrt{2}$				

19. (a) scalar (b) scalar (c) vector (d) vector (e) vector
(f) scalar (g) scalar (h) scalar (i) vector (j) scalar

20. (a) 1 (b) 0 (c) 1

21. (a) $a^2 - b^2$ (b) $a^2 + 2\mathbf{a}\cdot\mathbf{b} + b^2$ (c) $a^2 - 2\mathbf{a}\cdot\mathbf{b} + b^2$ (d) $4a^2 - b^2$ (e) $a^2 + \mathbf{a}\cdot\mathbf{b} - 6b^2$ (f) a^2

23. b, d **24.** a, b, d **25.** $x_1x_2 + y_1y_2$ **26.** 2·8 **27.** 0·72

28. c **29.** (a) 62° (b) 25 (c) 9 (d) 20 (e) $2\sqrt{5}$ **30.** 100

31. We can determine the scalar product of two vectors but not of a vector and a scalar.

Exercise 8B. Page 141.

1. (a) 3 (b) 3 (c) 8 (d) 4 **2.** (a) 7 (b) 14 (c) 14 (d) 18

3. (a) 8 (b) 48 (c) 22 (d) −12

4. (a) Not perpendicular (b) Not perpendicular (c) Perpendicular (d) Perpendicular
(e) Not perpendicular (f) Perpendicular

5. (a) 10 (b) −16 (c) 1 (d) −25 **6.** (a) 15 (b) 17 (c) 10 (d) −14

7. (a) −10 (b) 7 (c) $-3\mathbf{i}+\mathbf{j}$ (d) −3 **8.** (a) 5 (b) 13 (c) −33, 121°

9. (a) $7\sqrt{2}$ (b) 17 (c) 49, 73° **12.** (a) 24 (b) 16°

13. (a) 204 (b) 51° **14.** (a) 4 (b) 60°

15. (a) 0 (b) 90° **16.** (a) −75 (b) 180°

17. (a) 12 (b) 23° **18.** $\lambda = 8, \mu = 10{\cdot}5$

19. $w = 7, x = -5$ **20.** (a) $2\sqrt{5}$ (b) $4\mathbf{i}+2\mathbf{j}$ (c) 2 (d) $1{\cdot}2\mathbf{i}+1{\cdot}6\mathbf{j}$

21. $\pm(20\mathbf{i}+15\mathbf{j})$ **22.** $\pm\frac{1}{\sqrt{5}}(\mathbf{i}-2\mathbf{j})$ **23.** $5\mathbf{i}-2\mathbf{j}$, $-2\mathbf{i}-5\mathbf{j}$

24. (a) $4\mathbf{i}+2\mathbf{j}$ (b) $2\mathbf{i}-5\mathbf{j}$ (c) −2 (d) 95° **25.** −8 and 2

Exercise 8C. page 143.

1. (a) $\mathbf{b}-\mathbf{a}$ **2.** (a) 0 (b) $\mathbf{c}-\mathbf{a}$, $\mathbf{c}+\mathbf{a}$

4. (a) $\mathbf{c}-\mathbf{a}$, $\frac{1}{2}(\mathbf{a}+\mathbf{c})$ **5.** (a) $\mathbf{b}-\mathbf{c}$, $\mathbf{b}$, $\mathbf{b}+\mathbf{c}$

Miscellaneous Exercise Eight. Page 146.

1. If quadrilateral ABCD is a rhombus then it is a parallelogram but the converse is not true i.e. if quadrilateral ABCD is a parallelogram it does not have to be a rhombus. Hence the two way nature of the statement claimed by the use of the symbol "⇔" is not the case. The given statement is incorrect.

If PQRS is a rhombus then its diagonals will cut at right angles but the converse is not true i.e. if the diagonals of quadrilateral cut at right angles the quadrilateral is not necessarily a rhombus, it could be a kite for example. Hence the two way nature of the statement claimed by the use of the symbol "⇔" is not the case. The given statement is incorrect.

If the diagonals of a parallelogram cut at right angles then the parallelogram is a rhombus. This was proved in question 3 of Exercise 8B. Also if a shape is a rhombus then its diagonals will cut at right angles. This was proved in example 7 of chapter 8. The given statement is correct.

2.	a, b, d		**3.**	7i		**4.**	542640							
5.	(a)	$2\sqrt{29}$ units		(b)	$6\mathbf{i}+10\mathbf{j}$		(c)	$\mathbf{i}+8\mathbf{j}$						
6.	(a)	125		(b)	60		(c)	24	(d)	36		(e)	21	
7.	(a)	146°		(b)	4		(c)	9	(d)	3		(e)	$\sqrt{3}$	
8.	(a)	720		(b)	120		(c)	240	(d)	6				
	(e)	36		(f)	6		(g)	144						
9.	339°, 36 seconds													
10.	(a)	40 320		(b)	1 440		(c)	384						
11.	1 : 3, 1 : 4													
12.	(a)	154 440		(b)	63 000									
13.	(a)	$-6\mathbf{i}+\mathbf{j}$		(b)	$5\mathbf{i}+5\mathbf{j}$		(c)	−25	(d)	126°				
14.	$-\frac{14}{3}$ and 8		**15.**	6, 13·5		**16.**	$(8\mathbf{i}-3\mathbf{j})$ N, 54·2°							
17.	$h=\frac{6}{7}$, $k=\frac{2}{7}$		**18.**	(a)	60 480, all of them		(b)	60 480, 55 440 of them						
19.	462		**20.**	(a)	$\frac{1}{2}\mathbf{c} - h\mathbf{c} - \frac{1}{2}\mathbf{a}$		(b)	$\frac{1}{2}\mathbf{c} - k\mathbf{c} - \frac{1}{2}\mathbf{a}$						

Answers. Unit Two.

Exercise 9C. Page 166.

1. (a) $-\frac{24}{25}$ (b) $\frac{7}{25}$ (c) $-\frac{24}{7}$

2. (a) $\frac{120}{169}$ (b) $\frac{119}{169}$ (c) $\frac{120}{119}$

3. (a) $3 \sin 2A$ (b) $2 \sin 4A$ (c) $\frac{1}{2} \sin A$

4. (a) $2 \cos 4A$ (b) $1 \cos A$ (c) $1 \cos 4A$

5. (a) $-\frac{336}{625}$ (b) $\frac{527}{625}$ (c) $-\frac{336}{527}$

6. $15°$, $75°$, $195°$, $255°$

7. $-150°, -90°, -30°, 90°$

8. $0°, 75{\cdot}5°, 180°, 284{\cdot}5°, 360°$

9. $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

10. $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

11. $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}$

12. $66{\cdot}4°, 293{\cdot}6°, 426{\cdot}4°$

Exercise 9D. Page 169.

1. $5 \cos(\theta + 53{\cdot}1°)$

2. $13 \cos(\theta + 22{\cdot}6°)$

3. $5 \cos(\theta - 0{\cdot}64)$

4. $25 \cos(\theta - 1{\cdot}29)$

5. $13 \sin(\theta + 67{\cdot}4°)$

6. $25 \sin(\theta + 73{\cdot}7°)$

7. $5 \sin(\theta - 0{\cdot}64)$

8. $\sqrt{13} \sin(\theta - 0{\cdot}98)$

10. (a) $\sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right)$ (b) $\sqrt{2}, \frac{\pi}{4}$

11. $2{\cdot}09$, $6{\cdot}05$

12. $0{\cdot}33$, $1{\cdot}88$

13. $1{\cdot}36$, $5{\cdot}68$

Exercise 9E. Page 172.

1. $\frac{\pi}{3}, \frac{5\pi}{3}$

2. $\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

3. $0°, 70{\cdot}5°, 180°, 289{\cdot}5°, 360°$

4. $\pm 60°$

5. $63{\cdot}4°, 116{\cdot}6°, 243{\cdot}4°, 296{\cdot}6°$

6. $\frac{5\pi}{12}, \frac{23\pi}{12}$

7. $0°, 120°, 240°, 360°$

8. $3{\cdot}48$, $5{\cdot}94$

Exercise 9F. Page 176.

1. $\frac{1}{2}\cos 5x + \frac{1}{2}\cos x$

2. $\frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x$

3. $\frac{1}{2}\sin 8x + \frac{1}{2}\sin 6x$

4. $\frac{1}{2}\sin 4x - \frac{1}{2}\sin 2x$

5. $2 \cos 3x \cos 2x$

6. $-2 \sin 3x \sin 2x$

7. $2 \sin 4x \cos 2x$

8. $2 \cos 4x \sin x$

9. $\frac{2 + \sqrt{3}}{4}$

10. $\frac{\sqrt{6}}{2}$

11. $12°, 24°, 84°, 96°, 156°, 168°$

12. $0, \frac{\pi}{6}, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \frac{5\pi}{6}, \pi.$

13. $0°, 45°, 135°, 180°, 225°, 315°, 360°$

14. $-\pi, -\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi$

Exercise 9G. Page 180.

There are often many ways of writing the answers to these questions but all correct versions will generate the same set of solutions for $n \in \mathbb{Z}$. For some of the questions "common" alternatives are shown here.

1. $x = \begin{cases} 30° + n \times 360°, \\ 150° + n \times 360°, \end{cases}$ for $n \in \mathbb{Z}$.

2. $x = n \times 360°$, for $n \in \mathbb{Z}$.

3. $x = n \times 180° - 30°$, for $n \in \mathbb{Z}$. Could be written as $x = n \times 180° + 150°$, for $n \in \mathbb{Z}$.

4. $x = n \times 180° + 30°$, for $n \in \mathbb{Z}$.

5. $x = \begin{cases} 35{\cdot}2° + n \times 120°, \\ 4{\cdot}8° + n \times 120°, \end{cases}$ for $n \in \mathbb{Z}$. Could be written as $x = \begin{cases} 35{\cdot}2° + n \times 120°, \\ 124{\cdot}8° + n \times 120°, \end{cases}$ for $n \in \mathbb{Z}$.

6. $x = n \times 90° + 9{\cdot}3°$, for $n \in \mathbb{Z}$.

7. $x = \begin{cases} \frac{7\pi}{12} + n\pi, \\ \frac{11\pi}{12} + n\pi, \end{cases}$ for $n \in \mathbb{Z}$. Could be written as $x = \begin{cases} n\pi - \frac{\pi}{12}, \\ n\pi + \frac{7\pi}{12}, \end{cases}$ for $n \in \mathbb{Z}$.

8. $x = n\pi + \frac{\pi}{4}$, for $n \in \mathbb{Z}$.

9. $x = n\pi$, for $n \in \mathbb{Z}$.

10. $x = \begin{cases} \frac{2n\pi}{3} + \frac{\pi}{18}, \\ \frac{2n\pi}{3} + \frac{5\pi}{18}, \end{cases}$ for $n \in \mathbb{Z}$.

11. $x = \begin{cases} \frac{n\pi}{2} + 0{\cdot}84, \\ \frac{n\pi}{2} + 1{\cdot}16, \end{cases}$ for $n \in \mathbb{Z}$.

12. $x = n\pi \pm \frac{\pi}{6}$, for $n \in \mathbb{Z}$.

13. $x = \frac{n\pi}{3} + \frac{\pi}{4}$ for $n \in \mathbb{Z}$. Could be written as $x = \begin{cases} \frac{2n\pi}{3} + \frac{\pi}{4}, \\ \frac{2n\pi}{3} - \frac{\pi}{12}, \end{cases}$ for $n \in \mathbb{Z}$.

14. $x = \begin{cases} \frac{8n}{3} + 0{\cdot}44, \\ \frac{8n}{3} + 1{\cdot}56, \end{cases}$ for $n \in \mathbb{Z}$.

Exercise 9H. Page 182.

1. (a) $y = 3\sin x$ (b) $y = 4\sin x$

(c) $y = -3\sin x$ (d) $y = -4\sin x$

2. (a) $y = 3\sin 2x$ (b) $y = 4\sin\dfrac{2x}{3}$

(c) $y = 4\sin\left(\dfrac{2\pi}{5}x\right)$ (d) $y = -5\sin\left(\dfrac{\pi}{3}x\right)$

3. (a) $y = 2 + 3\sin x$ (b) $y = -2 - 4\sin x$

4. (a) $y = 3\sin\left(x - \dfrac{\pi}{2}\right)$ (b) $y = 4\sin\left(x + \dfrac{\pi}{2}\right)$

5. (a) $y = 5\sin\left(\dfrac{\pi}{4}(x - 2)\right)$ (b) $y = 4\sin\left(\dfrac{\pi}{5}(x - 3)\right)$

6. (a) $y = 3\sin\left(\dfrac{\pi}{4}(x - 1)\right) + 7$ (b) $y = 2\sin\left(\dfrac{\pi}{30}(x - 10)\right) + 7$

7. $h = 5\sin\left(\dfrac{2\pi}{365}t\right) + 12$

8. (a) $d = -6\cos\left(\dfrac{4\pi}{25}t\right) + 10$ (b) $d = 6\sin\left(\dfrac{4\pi}{25}\left(t - \dfrac{25}{8}\right)\right) + 10$

9. (a) $h = 3\cos\left(\dfrac{\pi}{3}(t - 2)\right) + 6$ (b) $h = 3\sin\left(\dfrac{\pi}{3}\left(t - \dfrac{1}{2}\right)\right) + 6$

Miscellaneous Exercise Nine. Page 186.

1. $\dfrac{\pi}{20}, \dfrac{3\pi}{20}, \dfrac{9\pi}{20}, \dfrac{11\pi}{20}, \dfrac{17\pi}{20}, \dfrac{19\pi}{20}.$

3. $30°, 120°, 210°, 300°$

4. $-\dfrac{\pi}{3}, 0, \dfrac{\pi}{3}.$

5. $x = \begin{cases} \dfrac{n\pi}{2} + \dfrac{\pi}{12}, \\ \dfrac{n\pi}{2} + \dfrac{\pi}{6}, \end{cases}$ for $n \in \mathbb{Z}$.

6. (a) $\sqrt{149}\sin(\theta - 0{\cdot}96)$ (b) $-\sqrt{149}$, $5{\cdot}67$

7. (a) "Eye-balling" the graph certainly suggests that a sinusoidal model could well be appropriate.
Taking the high of 27·2 and the low of 17·0 suggest an amplitude of 5·1.
Hence a= 5·1 and d = 22·1.
With a period of 12 units we have $\dfrac{2\pi}{b} = 12$. Hence $b = \dfrac{\pi}{6}$.
Thus $T = 5{\cdot}1\sin\left(\frac{\pi}{6}(x \pm ?)\right) + 22{\cdot}1$.
The typical "start" of "$y = a\sin x + b$" seems to have been moved right 10 units (or left 2 units).
Thus $T = 5{\cdot}1\sin\left(\frac{\pi}{6}(x - 10)\right) + 22{\cdot}1$. (Or: $T = 5{\cdot}1\sin\left(\frac{\pi}{6}(x + 2)\right) + 22{\cdot}1$).

Exercise 10A. Page 191.

1. $A_{4\times 2}$, $B_{2\times 4}$, $C_{4\times 1}$, $D_{4\times 3}$, $E_{2\times 2}$, $F_{1\times 3}$, $G_{3\times 2}$, $H_{4\times 4}$

2. (a) 4 (b) −4 (c) 7 (d) 7 (e) 3 (f) 0

3. (a) Cannot be determined (b) $\begin{bmatrix} 3 & -1 \\ 1 & -9 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -5 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix}$ (e) $\begin{bmatrix} 9 & -3 \\ 6 & 12 \\ 0 & 9 \end{bmatrix}$

(f) Cannot be determined (g) $\begin{bmatrix} 2 & 4 \\ 0 & -8 \end{bmatrix}$ (h) $\begin{bmatrix} 0 & 7 \\ -1 & -3 \end{bmatrix}$

4. (a) $\begin{bmatrix} 5 & 3 & -1 \\ 1 & 3 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 & 1 \\ -1 & -5 & -3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 6 & 3 \\ 6 & 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 4 & -3 \\ 3 & 14 & 9 \end{bmatrix}$

5. (a) Cannot be determined (b) $\begin{bmatrix} 6 & 12 \\ 3 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 3 & 11 \end{bmatrix}$ (d) Cannot be determined

6. (a) Cannot be determined (b) $\begin{bmatrix} 6 & 4 & 3 & 0 \\ 2 & 2 & 6 & 6 \\ 1 & 5 & 3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 2 & 8 \\ 4 & 2 & -6 \\ 0 & 2 & 4 \\ 2 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 14 & -3 & 6 \\ -2 & 4 & 6 & 12 \\ -1 & -5 & 3 & 20 \end{bmatrix}$

7. (a) No (b) No (c) Yes (d) Yes (e) Yes (f) No (g) Yes (h) No

8. Yes **9.** Yes **10.** $\begin{bmatrix} 1 & 2 & -3 \\ 1 & 0 & -2 \end{bmatrix}$

11. (a)
$$\begin{array}{l} \\ \text{Alan} \\ \text{Bob} \\ \text{Dave} \\ \text{Mark} \\ \text{Roger} \end{array} \begin{array}{c} \begin{array}{ccc} \text{P} & \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 40 & 20 & 4 \\ 37 & 15 & 14 \\ 47 & 19 & 9 \\ 39 & 21 & 3 \\ 39 & 19 & 16 \end{bmatrix} \end{array}$$

(b)
$$\begin{array}{l} \\ \text{Alan} \\ \text{Bob} \\ \text{Dave} \\ \text{Mark} \\ \text{Roger} \end{array} \begin{array}{c} \begin{array}{ccc} \text{P} & \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 10 & 5 & 1 \\ 9{\cdot}25 & 3{\cdot}75 & 3{\cdot}5 \\ 11{\cdot}75 & 4{\cdot}75 & 2{\cdot}25 \\ 9{\cdot}75 & 5{\cdot}25 & 0{\cdot}75 \\ 9{\cdot}75 & 4{\cdot}75 & 4 \end{bmatrix} \end{array}$$

12.
$$\begin{array}{l} \\ \text{Centre I} \\ \text{Centre II} \\ \text{Centre III} \\ \text{Centre IV} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{B} & \text{F} & \text{FL} & \text{G} & \text{GG} \end{array} \\ \begin{bmatrix} 6160 & 1925 & 2552 & 1947 & 4675 \\ 3124 & 1397 & 1507 & 1122 & 2992 \\ 5555 & 1617 & 3102 & 1408 & 2970 \\ 2409 & 1034 & 1672 & 924 & 1958 \end{bmatrix} \end{array}$$

13. $\begin{vmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{vmatrix}$ **14.** $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 8 & 16 \\ 3 & 9 & 27 & 81 \end{vmatrix}$

Exercise 10B. Page 197.

1. $\begin{bmatrix} 4 & 9 \end{bmatrix}$

2. Cannot be determined. Number of columns in 1st matrix ≠ number of rows in 2nd matrix.

3. $\begin{bmatrix} 2 & 10 \\ 1 & 4 \end{bmatrix}$ **4.** $\begin{bmatrix} 7 \end{bmatrix}$ **5.** $\begin{bmatrix} 3 & 1 \\ 12 & 4 \end{bmatrix}$

6. $\begin{bmatrix} 13 & -4 \\ -14 & 7 \end{bmatrix}$ **7.** $\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}$

9. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **10.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **11.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **13.** $\begin{bmatrix} 8 \end{bmatrix}$ **14.** $\begin{bmatrix} 3 & 2 & 3 \\ 4 & 3 & 1 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 0 & 5 \\ 10 & 2 & -2 \\ 6 & 1 & 4 \end{bmatrix}$ **16.** $\begin{bmatrix} 10 & 3 \\ 9 & 10 \end{bmatrix}$ **17.** $\begin{bmatrix} 14 \\ 32 \end{bmatrix}$

18. $\begin{bmatrix} 2 & 4 & 1 \\ 5 & 7 & 18 \\ 12 & 8 & 22 \end{bmatrix}$

19. (a) $\begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 2 & 3 \\ 4 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 & 2 \\ 2 & 3 & 4 \\ -2 & -2 & 1 \end{bmatrix}$

20. No. Justify by showing example for which $AB \neq BA$.

24. (a) Cannot be formed (b) Cannot be formed (c) 3×3 (d) 2×2
(e) Cannot be formed (f) 1×2 (g) 3×2 (h) 1×3

25. (a) Yes (b) Yes (c) Yes (d) No (e) No (f) No (g) No (h) Yes

26. Matrix A must be a square matrix.

27. AA, AC, BA, CB.

28. (a) $\begin{bmatrix} -1 & -2 \\ 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -2 \\ 7 & -3 \end{bmatrix}$

29. (a) 1^{st} B, 2^{nd} E, 3^{rd} C, 4^{th} D, 5^{th} A.
(b) 1^{st} =B & C, 3^{rd} E, 4^{th} D, 5^{th} A.

30. Initially: $\begin{array}{l} \text{Client 1} \\ \text{Client 2} \\ \text{Client 3} \end{array} \begin{bmatrix} \$15000 \\ \$15000 \\ \$15000 \end{bmatrix}$ Two years later: $\begin{array}{l} \text{Client 1} \\ \text{Client 2} \\ \text{Client 3} \end{array} \begin{bmatrix} \$17700 \\ \$19300 \\ \$18800 \end{bmatrix}$

31. $\begin{array}{cc} \text{Drink (mLs)} & \text{Burgers} \\ \end{array}$
$\begin{bmatrix} 18125 & 55 \end{bmatrix}$

32. (a) QP (b) $\begin{array}{ccc} \text{Hotel A} & \text{Hotel B} & \text{Hotel C} \end{array}$
$\begin{bmatrix} \$4610 & \$3680 & \$2665 \end{bmatrix}$
Displays total nightly tariff for each hotel when full.

(c) Row 1 column 1 of PR would be
Single rooms in A × Single room tariff +
Single rooms in B × Double room tariff +
Single rooms in C × Suite tariff
Thus PR not giving useful information.

33. (a) $\begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ (b) $\begin{array}{cccc} \text{Poles} & \text{Decking} & \text{Framing} & \text{Sheeting} \\ 25 & 205 & 145 & 320 \end{array}$

Matrix shows number of metres of each size of timber required to complete order.

(c) $\begin{bmatrix} \$4 \\ \$2 \\ \$3 \\ \$1{\cdot}50 \end{bmatrix}$ Product will have dimensions 3 ×1. Matrix will display the total cost of timber for each type of cubby.

34. (a) $E = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 800 & 50 & 1000 \end{bmatrix} \end{array}$ (b) $\begin{array}{c} \begin{array}{cccc} \text{Model I} & \text{Model II} & \text{Model III} & \text{Model IV} \end{array} \\ \begin{bmatrix} 4600 & 4900 & 6300 & 5600 \end{bmatrix} \end{array}$

Matrix displays the total cost of commodities, in dollars, for each model type.

35. (a) RP (b) $\begin{bmatrix} 6700 & 7200 & 2300 \end{bmatrix}$ (c) Matrix shows the number of minutes required for cutting (6700 minutes) assembling (7200 minutes) and packing (2300 minutes) to complete the order.

Exercise 10C. Page 207.

1. -2 **2.** 10 **3.** 7 **4.** 7

5. 5 **6.** 0 **7.** $-x^2$ **8.** $x^2 - y^2$

9. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ **10.** $\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$ **11.** $\frac{1}{3}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$ **12.** $\frac{1}{5}\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$

13. $\frac{1}{10}\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$ **14.** $\frac{1}{10}\begin{bmatrix} -3 & -1 \\ 1 & -3 \end{bmatrix}$ **15.** Singular **16.** Singular

17. Singular **18.** $\frac{1}{x}\begin{bmatrix} 1 & -y \\ 0 & x \end{bmatrix}, x \neq 0$ **19.** $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **20.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

21. (a) True (b) True (c) Not necessarily (d) True (e) True
(f) True (g) True (h) True (i) Not necessarily (j) Not necessarily

22. $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ **23.** $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **24.** $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$

25. $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ **26.** 7 **27.** 5

28. (a) $\begin{bmatrix} -13 & 4 \\ 12 & -4 \end{bmatrix}$ (b) 10 (c) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ (d) $\frac{1}{10}\begin{bmatrix} 5 & 5 \\ 14 & 16 \end{bmatrix}$ (e) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ (f) $\begin{bmatrix} 6 & 1 \\ -4 & 1 \end{bmatrix}$

29. (a) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$ (c) $\frac{1}{6}\begin{bmatrix} 1 & -2 \\ 0 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ (e) $\begin{bmatrix} 6 & -2 \\ -3 & 2 \end{bmatrix}$

30. $\begin{bmatrix} 2 & -1 \\ 17 & -9 \end{bmatrix}$ **31.** $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **32.** (a) 8 (b) ±4 (c) −4 or 5

33. $F = \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}$, $G = \begin{bmatrix} 3 & -4 \\ 0 & 2 \end{bmatrix}$ **34.** $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 1 & 4 & 1 \end{bmatrix}$

35. $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

36. (a) $\begin{bmatrix} \$24 & \$56 \\ \$16 & \$36 \end{bmatrix}$ (b) $\begin{bmatrix} 0{\cdot}8 & 0 \\ 0 & 1 \end{bmatrix}$

37. $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$

38. $\begin{bmatrix} 11 & 20 \\ 5 & 7 \end{bmatrix}$

39. $\begin{bmatrix} -1 & 0 \\ -15 & 2 \end{bmatrix}$

40. (a) $\begin{bmatrix} 6 & 5 \\ 8 & 7 \end{bmatrix}$ (b) $BA = \begin{bmatrix} 860 & 740 \end{bmatrix}$ i.e. $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 8 & 7 \end{bmatrix}\begin{bmatrix} 860 & 740 \end{bmatrix}$

(c) $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 860 & 740 \end{bmatrix} A^{-1}$ giving $x = 50$ and $y = 70$.

Exercise 10D. Page 2012.

1. $\begin{bmatrix} 2 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 2 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & 2 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 0 \\ 2 & 0 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -3 \\ 0 & -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

7. (a) $\frac{1}{2}\begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$ (b) $x = -1,\ y = -3{\cdot}5$

8. (a) $\begin{bmatrix} -2{\cdot}5 & -2 & 0{\cdot}5 \\ -2 & -2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (b) $x = -1,\ y = 5,\ z = 2$

9. (a) $x = 3, y = -7$ (b) $x = 5{\cdot}5, y = -8{\cdot}5$

10. (a) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ (b) $A^{-1} = \frac{1}{7}B$ (c) $x = 3, y = -1, z = 1$

11. (a) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 2 & -1 \\ 2 & -1 & 3 & -1 & 2 \\ 3 & 2 & -1 & -1 & -2 \\ 0 & 2 & 0 & 3 & -1 \end{bmatrix}\begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 13 \\ 2 \\ 4 \\ 8 \end{bmatrix}$ (b) $v = 1,\ w = -1,\ x = 3,\ y = 2,\ z = -4.$

Miscellaneous Exercise Ten. Page 214.

1. (a) $B = \begin{bmatrix} -2 & 0 \\ 4 & 3 \end{bmatrix}$ (b) $C = \begin{bmatrix} -1 & 0 \\ 4 & 4 \end{bmatrix}$

2. (a) $E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $F = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$ (c) $G = \begin{bmatrix} 6 & -1 \\ 2 & 1 \end{bmatrix}$

(d) $H = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$ (e) $K = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$

3. $\frac{\pi}{12}, \frac{5\pi}{12}$

4. $\theta = 0,\ p,\ \pi,\ (\pi + p),\ 2\pi$

7. (a) $2y^2 + y - 1$ (b) $-\frac{11\pi}{6}, -\frac{7\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

8. (a) $\sqrt{29}\cos(\theta - 68{\cdot}2°)$ (b) $-\sqrt{29}$, $248{\cdot}2°$

9. (a) Cannot be determined. A and B are not the same size and so A + B cannot be formed.

(b) $\begin{bmatrix} 0 & -1 & 1 \\ 5 & -2 & 5 \end{bmatrix}$

(c) Cannot be determined. The number of columns in A ≠ the number of rows in B.

(d) $\begin{bmatrix} 5 & -2 & 8 \\ 2 & -1 & 3 \end{bmatrix}$

(e) Cannot be determined. The number of columns in A ≠ the number of rows in C.

(f) $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

10. (a) XY (b) $\begin{bmatrix} 420 \\ 410 \\ 430 \end{bmatrix}$ (c) $\begin{bmatrix} \text{Commodity cost (\$) to produce one model A} \\ \text{Commodity cost (\$) to produce one model B} \\ \text{Commodity cost (\$) to produce one model C} \end{bmatrix}$

11. Of the four products in the list only XZ can be formed.
It shows the total points obtained by each team.

$$XZ = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \text{Points} \\ \begin{bmatrix} 13 \\ 10 \\ 9 \\ 10 \\ 15 \end{bmatrix} \end{array}$$

12. (a) $y = 4\sin 8x$ (b) $y = -3\sin\left(\frac{2\pi}{5}x\right)$

13. (b) $y = 2\sin\left(\frac{\pi}{2}(x-1)\right)$ (b) $y = 20\sin\left(\frac{\pi}{15}(x+5)\right)$

14. (b) $y = 5\sin\left(\frac{\pi}{5}(x-2)\right) + 10$ (b) $y = 10\sin\left(\frac{\pi}{50}(x-20)\right) + 40$

15. $x = -1, y = -2, p = -5, q = 7, r = -7, s = 2.$

16. $\begin{bmatrix} -1 & 1 \\ 3 & -5 \end{bmatrix}$

Exercise 11A . Page 219.

1. Rotate 180° about origin.
2. Rotate 90° anticlockwise about origin.
3. Reflect in the x-axis.
4. Reflect in the y-axis.
5. Reflect in the line $y = x$.
6. Reflect in the line $y = -x$.
7. Dilation parallel to x-axis, scale factor 2.
8. Dilation parallel to y-axis, scale factor 3.
9. Dilation parallel to x-axis, scale factor 2 and dilation parallel to y-axis, scale factor 3.
10. Dilation parallel to x-axis, scale factor 3 and dilation parallel to y-axis, scale factor 3.
11. Shear parallel to x-axis, scale factor 2.
12. Shear parallel to y-axis, scale factor 3.
13. Results of (a) and (b) should lead to conclude $\dfrac{\text{Area O'A'B'C'}}{\text{Area OABC}} = |\text{determinant of matrix}|$.

Exercise 11B. Page 223.

1. (a) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

2. (a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
3. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
4. $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

5. (a) $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 1 & -2 & -5 & -5 \end{bmatrix}$ (b) A'(0, 1), B'(1, −2), C'(3, −5), D'(2, −5)

6. A(1, 3), B(1, 1), C(4, −3)
7. A(1, 3), B(−1, 2), C(0, 2)
8. $\begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$
10. $\begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$
11. $\begin{bmatrix} 0 & 1 \\ -1 & 3 \end{bmatrix}$

12. a = 2, b = 5, c = 1, d = 3
13. (a) $\begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 9 \\ -1 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$
15. (a) 36 square units (b) O'(0, 0), A'(12, 3), B'(8, 5), C'(−4, 2) (c) Diagram not shown here.
16. (a) Diagram not shown here. (b) 8 square units (c) 40 square units
(d) Diagram, not shown here, should have A'(−2, 2), B'(−4, −6), C'(2, −2), D'(4, 6).

18. $y = 3x - 1$
20. (a) (10, 5) (b) $y = 0{\cdot}5x$
21. $m_2 = \dfrac{m_1 + 2}{3}$. $-3 + \sqrt{10}$ and $-3 - \sqrt{10}$

Exercise 11C. Page 228.

1. (a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

2. (a) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (c) A repeat reflection will return us to the original position. Hence the square of each matrix is the identity because the repeat reflection leaves the final position identical to initial position.

3. $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
6. $\alpha = 2(\phi - \theta)$

7. (a) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 6 \\ 4 \end{bmatrix}$ (b) $\dfrac{1}{\sqrt{13}}\begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$
(c) $O''(2\sqrt{13}, 0)$, $A''\left(\frac{23\sqrt{13}}{13}, \frac{2\sqrt{13}}{13}\right)$, $B''\left(\frac{21\sqrt{13}}{13}, -\frac{\sqrt{13}}{13}\right)$, $C''\left(\frac{24\sqrt{13}}{13}, -\frac{3\sqrt{13}}{13}\right)$.

Miscellaneous Exercise Eleven. Page 229.

2. $0,\ \pi,\ \frac{7\pi}{6},\ \frac{11\pi}{6},\ 2\pi$ **3.** $a = 4, b = 0, c = -3, d = 0$

4. (a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $|\det| = 1.$ ✓ (b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $|\det| = 1.$ ✓ (c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $|\det| = 1.$ ✓

(d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $|\det| = 1.$ ✓ (e) $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$, $|\det| = 1.$ ✓ (f) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, $|\det| = 1.$ ✓

5. BAC, $\begin{bmatrix} 5 & 5 & 2 & 4 \end{bmatrix}$

6. (a) Cannot be determined. A and B are not of the same size.

(b) $\begin{bmatrix} 3 & 1 \\ 0 & 5 \end{bmatrix}$

(c) Cannot be determined. Number of columns in A ≠ Number of rows in C.

(d) $\begin{bmatrix} 1 & 3 & 0 \\ 1 & -3 & -2 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 \\ -2 & 6 \end{bmatrix}$ (f) $\begin{bmatrix} 5 & 5 \\ 5 & 10 \end{bmatrix}$

(g) Cannot be determined. BA can be formed but cannot be added to C as of different size.

7. To be singular we require determinant to be zero. For given matrix, determinant = $2x^2 + 4$ which is ≥ 4 for all real x. Thus determinant cannot be zero for real x. Thus not a singular matrix.

8. k = 3, p = 12, q = −9

9. (a) $\begin{bmatrix} 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -4 & 4 \\ 0 & 0 & 0 \\ -1 & 2 & -2 \end{bmatrix}$ **10.** $x = 5, y = -2$

11. y must equal zero, no restrictions necessary on x and z.

12. a = 4, b = 3·5, c = −2, d = −0·5 **13.** (a) $\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$ **15.** $x = \frac{n\pi}{2} + 2{\cdot}08$ for $n \in \mathbb{Z}$.

Exercise 12A. Page 233.

1 to 10. Answers not given here. Compare your answers with those of others in your class.

11. (a) $\frac{5}{9}$ (b) $\frac{25}{33}$ (c) $\frac{7}{11}$ (d) $\frac{743}{333}$ (e) $\frac{2083}{9000}$

12. If we assume that $\sqrt{2}$ can be written in the form a/b for integer a and b, b ≠ 0, and a and b having no common factors then $\sqrt{2} = \frac{a}{b}$

It therefore follows that $2 = \frac{a^2}{b^2}$

and so $2b^2 = a^2$

Thus a^2, and hence a, is even.

We could therefore write a as 2k, k an integer.

Hence
$$2b^2 = (2k)^2$$
$$2b^2 = 4k^2$$
$$b^2 = 2k^2$$

and so b^2, and hence b, is even.

But if a and b are both even they have a common factor, 2. Hence we have a contradiction.

Our original premise, or underlying assumption, about $\sqrt{2}$ must be false.

Hence $\sqrt{2}$ is irrational.

Exercise 12B. Page 234.

5. Yes always a multiple of ten. Compare your justification with others in your class.
No, not always a multiple of twenty. Justify using a counter example.

7. John's conjecture is not correct. $6^3 - 6 = 210$ which is not divisible by 12.
A possible alternative conjecture:

For any integer x, $x \geq 2$, $x^3 - x$ is always divisible by 6. (Proof not given here.)

Miscellaneous Exercise Twelve. Page 242.

1. (a) Cannot be determined (b) $\begin{bmatrix} -1 \\ 7 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -3 \\ -3 & 3 & -3 \end{bmatrix}$

(d) Cannot be determined (e) $\begin{bmatrix} 0 & 1 & 2 \\ 6 & 5 & 4 \end{bmatrix}$

2. $B = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$, $D = \begin{bmatrix} 5 & 4 \end{bmatrix}$, $E = \begin{bmatrix} 1 & -3 \end{bmatrix}$.

3. $\begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$

4. (a) XY, ZX (b) ZX (c) $\begin{array}{cc} \text{N}^{\text{o.}}\text{ of Aus stamps required} & \text{N}^{\text{o.}}\text{ of RoW stamps required} \\ \left[\begin{matrix} 18150 \end{matrix}\right. & \left.\begin{matrix} 14850 \end{matrix}\right] \end{array}$

5. $p = 0, q = 12, x = -3$

8. (a) $\sqrt{34}\cos(\theta + 0{\cdot}54)$ (b) $-\sqrt{34}$, $2{\cdot}60$

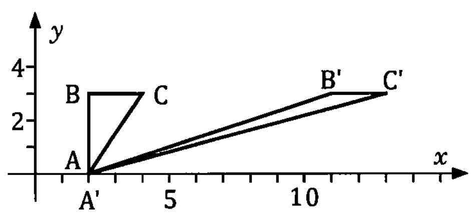
9. BAC, $\begin{bmatrix} 10 & 0 & 10 & 10 \\ 8 & 0 & 8 & 8 \end{bmatrix}$

10. Either $x = -3, y = 6, p = 0$ and $q = 24$ or $x = 2, y = -4, p = 0$ and $q = 24$.

11. No conflict. Final proof showing A = B is quite correct **provided A^{-1} exists.** In example 1, matrix A is not a square matrix so A^{-1} does not exist. In example 2, det A = 0, so A^{-1} does not exist.

12. AC, AD, BD, CB, DC, DD

13. A'(2, 0), B'(11, 3), C'(13, 3)
A shear parallel to x-axis, scale factor 3.



15. $x = \begin{cases} 2n\pi + 0{\cdot}64, \\ 2n\pi + 2{\cdot}21, \end{cases}$ for $n \in \mathbb{Z}$.

Exercise 13A. Page 249.

1. 5i **2.** 12i **3.** 3i

4. 7i **5.** 20i **6.** $\sqrt{5}\,i$

7. $2\sqrt{2}\,i$ **8.** $3\sqrt{5}\,i$ **9.** (a) 3 (b) 5

10. (a) −2 (b) 7 **11.** (a) 3 (b) −1 **12.** $-1 + 2i, -1 - 2i$

13. $-1 + \sqrt{2}\,i, -1 - \sqrt{2}\,i$ **14.** $-2 + \sqrt{2}\,i, -2 - \sqrt{2}\,i$ **15.** $-1 + 3i, -1 - 3i$

16. $2 + \sqrt{2}\,i, 2 - \sqrt{2}\,i$ **17.** $\frac{1}{4} + \frac{\sqrt{7}}{4}i, \frac{1}{4} - \frac{\sqrt{7}}{4}i$ **18.** $-\frac{1}{4} + \frac{\sqrt{7}}{4}i, -\frac{1}{4} - \frac{\sqrt{7}}{4}i$

19. $-\frac{3}{2} + \frac{1}{2}i, -\frac{3}{2} - \frac{1}{2}i$ **20.** $\frac{1}{2} + \frac{7}{2}i, \frac{1}{2} - \frac{7}{2}i$ **21.** $\frac{1}{5} + \frac{8}{5}i, \frac{1}{5} - \frac{8}{5}i$

22. $\frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$ **23.** $\frac{3}{10} + \frac{\sqrt{11}}{10}i, \frac{3}{10} - \frac{\sqrt{11}}{10}i$

Exercise 13B. Page 252.

1. $7 + 2i$ **2.** $3 - 10i$ **3.** $-3 + 4i$ **4.** $7 - 2i$

5. $-3 + 2i$ **6.** $7 - 2i$ **7.** $13 + 4i$ **8.** $12 + 7i$

9. $13 + 2i$ **10.** $7 + 8i$ **11.** $3 - 8i$ **12.** $10 - 15i$

13. 7 **14.** 5 **15.** $-4 + 19i$ **16.** $-3 + 11i$

17. $3 - i$ **18.** $-13 + 13i$ **19.** $\frac{1}{2} - \frac{1}{2}i$ **20.** $\frac{1}{5} + \frac{7}{5}i$

21. $\frac{2}{5} - \frac{6}{5}i$ **22.** $\frac{8}{17} + \frac{2}{17}i$ **23.** $0 + 1i$ **24.** $\frac{17}{13} - \frac{7}{13}i$

25. (a) $9 + i$ (b) $1 - 5i$ (c) $7 - 12i$ (d) $26 + 7i$ (e) $7 + 24i$ (f) $\frac{14}{25} - \frac{23}{25}i$

26. (a) 4 (b) $-2 - 10i$ (c) $6 - 10i$ (d) $28 - 10i$ (e) $-16 + 30i$ (f) $-\frac{11}{13} + \frac{10}{13}i$

27. (a) $24 + 7i$ (b) 48 (c) 625 (d) $\frac{527}{625} - \frac{336}{625}i$

28. (a) $4 - 9i$ (b) $18i$ (c) $20 - 9i$ (d) $-4 + 45i$ (e) 97 (f) $-\frac{65}{97} + \frac{72}{97}i$

29. $c = 3, d = 2$ **30.** $a = -5, b = -12$

31. $c = -10, d = 4$ **32.** $a = 15, p = 78$

33. (a) Yes, statement is correct for all complex z and w.

(b) No, eg $z = 3 + 2i$ and $w = 5 - 2i$: $\text{Im}(z) = -\text{Im}(w)$ but $w \neq \bar{z}$.

34. (a) $(x - 2 - 3i)(x - 2 + 3i)$ (b) $(x - 1 - 3i)(x - 1 + 3i)$

(c) $(x - 3 - 2\sqrt{2})(x - 3 + 2\sqrt{2})$ (d) $(x + 5 + i)(x + 5 - i)$

(e) $(x + 7 - 2i)(x + 7 + 2i)$ (f) $(x + 2 + \sqrt{7})(x + 2 - \sqrt{7})$

35. (b) $b = -6, c = 13$ (c) $d = -10, e = 34$ **36.** (a) -1 (b) i (c) i

37. $0{\cdot}25, 4$ and $-1, -1$ **38.** (a) 1

40. (a) $(2, 3)$ (b) $(-5, 6)$ (c) $(0, 7)$ (d) $(3, 0)$ (e) $(1, 9)$ (f) $(6, 0)$

(g) $(3, 3)$ (h) $(0, 14)$ (i) $(-10, 6)$ (j) $(10, 0)$ (k) $(0{\cdot}3, 0{\cdot}6)$ (l) $(-\frac{55}{73}, -\frac{48}{73})$

41. $-\frac{5}{53} - \frac{9}{53}i$

Exercise 13C. Page 256.

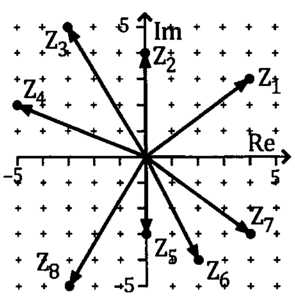
1. $Z_1 = 7 + 2i$ $Z_2 = 2 + 4i$ $Z_3 = 0 + 6i$ $Z_4 = -5 + 3i$ $Z_5 = -7 - 5i$

$Z_6 = 0 - 4i$ $Z_7 = 3 - 6i$ $Z_8 = 6 - 3i$

2. $Z_1 = (6, 0)$ $Z_2 = (7, 5)$ $Z_3 = (-3, 6)$ $Z_4 = (-5, 0)$ $Z_5 = (-6, -3)$

$Z_6 = (-3, -6)$ $Z_7 = (0, -6)$ $Z_8 = (7, -7)$

3.



4.

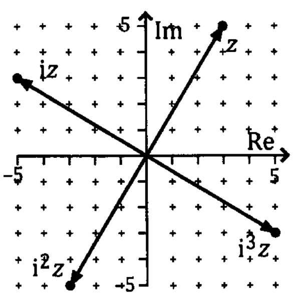
$Z_1 = 1 + 2i$

$Z_2 = -3 - 2i$

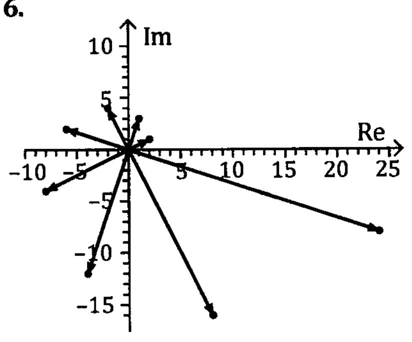
$Z_3 = -3 + 2i$

$Z_4 = 3 - 2i$

5.



6.



Miscellaneous Exercise Thirteen. Page 257.

1. (a) 29 (b) 10 (c) 40 (d) $-7 + 24i$ (e) $\frac{3}{10} - \frac{11}{10}i$ (f) $\frac{3}{13} + \frac{11}{13}i$

2. (a) $-1 + 2i$ (b) $9 + 19i$ (c) $2 + 3i$ (d) $9 - 19i$ (e) $-5 - 12i$ (f) $-280 + 342i$ (g) $2 - 5i$

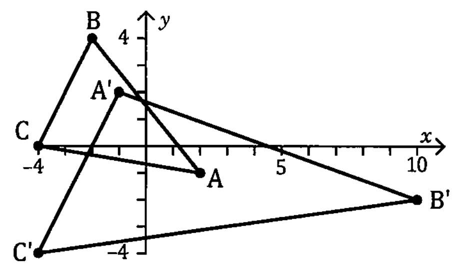
3. (a) $p = 2,\ q = 1,\ r = -2$ (b) $0{\cdot}5,\ 1 + \sqrt{2}\,i,\ 1 - \sqrt{2}\,i$

4. (a) 10 (b) 22

5. (a) $-5\sqrt{2}\,i$ (b) -50 (c) $-49 + 10\sqrt{2}\,i$

6. $a = 3$ and $b = -2$, $a = -3$ and $b = 2$

7. (a) $(2, -1)$, $(-2, 4)$, $(-4, 0)$ (b)



Hint for part (b):
To find the area of each triangle draw a rectangle around each one, find the area of the rectangle and subtract appropriate areas. Then confirm Area $\Delta A'B'C' = |\det T|$ Area ΔABC.

8. $a = 3,\ b = -1,\ c = 2,\ d = 1$

9. $2 - 2i,\ 2 - 12i$

10. (a) Areas multiplied by zero. Thus determinant equals zero.
(b) (0, 0) has image (0, 0). Thus (0, 0) lies on the line. Thus line passes through origin.

11. $a = 3,\ b = -5$

12. (a) $-119 - 120i$ (b) 12

13. $x^2 - 4x + 13 = 0$

14. $1 + 5i$

15. $\frac{7}{13} + \frac{17}{13}i$

16. $-5 + 3i,\ 16 - 30i$

18. $0°, 60°, 120°, 180°, 240°, 300°, 360°$

20. (a) $\begin{bmatrix} -12 & 20 \\ -3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -7 \end{bmatrix}$

21. (a) B (b) B, D (c) A, B, F (d) A, C (e) A, B, C, D (f) A, C (g) E (h) A, B, F

22. (a) BC^2B^{-1} (b) BC^3B^{-1} (c) BC^nB^{-1}

27. $x = n\pi + \frac{\pi}{12}$ for $n \in \mathbb{Z}$, $n\pi + \frac{5\pi}{12}$ for $n \in \mathbb{Z}$, $n\pi + \frac{\pi}{2}$ for $n \in \mathbb{Z}$.

28. (a) $\frac{1}{2}\begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$ (c) $\frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$

(d) $\frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} \times \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} \neq \frac{1}{2}\begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$

Rotating square 1 anticlockwise 30° about the origin makes A go to C" and C go to A".
So while the rotated image occupies the same space as square 3, it is not the same image because A does not go to A" and C does not go to C".